Grade 2 Mathematics, Quarter 4, Unit 4.1

Problem Solving With Money

Overview

Number of instructional days: 6 (1 day = 45–60 minutes)

Content to be learned

- Count dollar bills, quarters, dimes, nickels and pennies.
- Use $ and cent sign appropriately.
- Solve word problems involving putting together, taking apart and comparing.

Mathematical practices to be integrated

Look for and make use of structure.

- Look for and describe a pattern.
- Apply patterns to count money.

Attend to precision.

- Use either a dollar sign or a cent sign.
- Position dollar sign and cent sign appropriately.

Essential questions

- How can you find all the different ways to make _____? (a given monetary value)
- How do you know when your answer is reasonable?
- Which strategy do you use to count a given set of coins?
- How do you know when to use the $ sign or cent sign?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	2.MD

Work with time and money.

2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Money has not been addressed in prior grades.

Current Learning

Second graders count coins and bills to solve word problems. Word problems are a variety of types (put together, take apart, compare) with unknowns in different positions. Second graders use what they know about counting by ones, fives, tens, and 25s to count coins and bills. The expectation is not that second graders write money in a decimal format. They notate money using either a dollar sign or a cent sign.

According to Bloom's taxonomy working with coins and bills is developmental.

Routines: Tell time to the nearest five minutes. Count the number of units in an array and express the total as a sum of equal addends.

Future Learning

In grades 3, 4 and 5 students will work with fractions with 10 or 100 in the denominator. These fractions are called decimal fractions. Their work with decimal fractions is connected to money.

Additional Findings

Benchmark for Science Literacy states, "By the end of second grade, students should know that money can buy things that people need or want. People earn money by working at a job making or growing things, selling things, or doing things to help other people" (p. 168).

The book also states, "From the start, children study numbers and shapes and simple operations on them and do so in as many different contexts as possible. By the end of second grade, students should know… numbers can be used to count any collection of things" (p. 26).

According to *Principles and Standards for School Mathematics*, "K–2 teachers should help students recognize that solving one kind of problem is related to solving another kind of problem" (p. 83).

The book also states, "In pre–K through the 2nd grade, students should understand numbers, ways of representing numbers, relationships among numbers, and number systems; [they should] count with understanding and recognize, 'how many in sets of objects'" (p. 83).

A Research Companion to Principles and Standards for School Mathematics states, "Experience with addition, subtraction, multiplication, and division situations and with the language involved in them allows students to build a mathematically adequate understanding of the operations" (p. 69).

Grade 2 Mathematics, Quarter 4, Unit 4.2

Use Place Value, Composition, and Decomposition to Solve Addition Problems

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Add and subtract within 1000.
- Add up to four two-digit numbers.
- Use composition and decomposition as a strategy to join numbers.
- Use concrete models and drawings to represent thinking.
- Use place value as a strategy to join numbers.
- Use properties of the operation of addition to add numbers.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Analyze given information (including the specific numbers presented in the problem) to develop strategies for solving problems.
- Identify and execute appropriate strategies to solve the problem.
- Evaluate progress toward the solution and make revisions if necessary
- Ask, "Does this make sense?"

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Use varied representations and approaches when solving problems.
- Know and flexibly use different properties of operations and objects.
- Change perspectives, generate alternatives and consider different options.

Attend to precision.

- Communicate their understanding of mathematics to others.
- Strive for accuracy.

Essential questions

- How can you solve addition problems?
- Can you solve a given 2-digit addition problem in more than one way?
- What strategies do you use to add more than two 2-digit numbers?
- How can you communicate your thinking (about a given problem) using words, numbers, or pictures?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	2.NBT

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7 Add ~~and subtract~~ within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition ~~and subtraction;~~ relate the strategy to a written method. Understand that in adding ~~or subtracting~~ three-digit numbers, one adds ~~or subtracts~~ hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose ~~or decompose~~ tens or hundreds.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In first grade, students began to view 2-digit numbers as being composed of tens and ones. They used this knowledge to solve problems within 100 with understanding. They used concrete objects or drawings to make connections with written numerical work.

Current Learning

In this quarter second graders are using what they know about place value (100s, 10s, and ones) as well as the properties of the operation of addition such as the commutative and distributive property to compute addition problems up to 1000. They can decompose numbers in multiple ways so that 27 could be decomposed into 20 and 7 (place value) or into 25 and 2(or any other combination) depending upon the context. They record their thinking in a way that makes sense to them (helps them keep track so they are accurate in their computation) and communicates their thinking clearly to others. The expectation of the Common Core Standards is that students in second grade are using a variety of strategies to solve these problems but are not expected to be fluent in the use of a standard algorithm until the end of Grade 4.

According to Bloom's taxonomy, this work is both developmental and reinforcement.

Routines: Mentally add or subtract 10 or 100 to a given number between 100 and 900. Tell time to the nearest five minutes. Continue to work with arrays.

Future Learning

In grades 3, students will continue to add and subtract within 1000. They achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction. This is preparation for learning standard algorithms in grade 4.

Additional Findings

Principles and Standards for School Mathematics states, “The decisions that teachers make about problem solving opportunities influence the depth and breadth of students’ mathematics learnig. Teachers must be clear about the mathematics they want their students to accomplish as they structure situation that are both problematic and attainable for a wide range of students.” (pp118-119)

According to *A Research Companion to Principles and Standards for School Mathematics,* “An important distinction should be made between the situation representation (ie. An equation or drawing) and a solution representation. The most powerful problem solving approach is to understand the situation deeply—that is, to be able to draw it or otherwise represent it to oneself.” (p. 69)

“In Grade 2, students extend their understanding of the base-ten system by viewing 10 tens as forming a new unit called a “hundred”. This lays the groundwork for understanding the structure of the base-ten system as based in repeated bundling in groups of 10 and understanding that the unit associated with each place is 10 of the unit associated with the place to its right.

At grade 2 composing and decomposing involves an extra layer of complexity beyond that of grade 1. This complexity manifests itself in two ways. First, students must understand that a hundred is a unit composed of 100 ones, but also that it is composed of 10 tens. Second, there is the possibility that both a ten and a hundred are composed or decomposed. For example, in computing 398 + 7 a new ten and a new hundred are composed. In computing 302 – 184, a ten and a hundred are decomposed.

Students may continue to develop and use special strategies for particular numerical cases or particular problem situations such as Unknown Addends. *Progressions, Number and Operations in Base Ten* (pp. 8–9)

Grade 2 Mathematics, Quarter 4, Unit 4.3

Use Place Value, Composition, and Decomposition to Solve Subtraction Problems

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Subtract within 100.
- Use composition and decomposition as a strategy to find differences.
- Use concrete models and drawings to represent thinking.
- Use place value as a strategy to find differences between numbers.
- Use the relationship of addition and subtraction to solve subtraction problems.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Analyze given information (including the specific numbers presented in the problem) to develop strategies for solving problems.
- Identify and execute appropriate strategies to solve the problem.
- Evaluate progress toward the solution and make revisions if necessary
- Ask "Does this make sense?"

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Use varied representations and approaches when solving problems.
- Know and flexibly use different properties of operations and objects.
- Change perspectives, generate alternatives and consider different options.

Attend to precision.

- Communicate their understanding of mathematics to others.
- Strive for accuracy.

Essential questions

- How can you solve subtraction problems?
- What strategies will you use to solve a given 2-digit subtraction problem in more than one way?
- How can you communicate your thinking (about a given problem) using words, numbers, or pictures?
- What is 10 less/100 less than this number? Explain your thinking.

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	**2.NBT**

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In first grade, students began to view 2-digit numbers as being composed of tens and ones. They used this knowledge to solve problems within 100 with understanding. They used concrete objects or drawings to make connections with written numerical work.

Current Learning

In this quarter second graders are using what they know about place value (100s, 10s, and ones) as well as the relationship between addition and subtraction. Students may approach subtraction problems as missing addends. They can decompose numbers in multiple ways so that 27 could be decomposed into 20 and 7 (place value) or into 25 and 2(or any other combination) depending upon the context. They record their thinking in a way that makes sense to them (helps them keep track so they are accurate in their computation) and communicates their thinking clearly to others. The expectation of the Common Core Standards is that students in second grade are using a variety of strategies to solve these problems but are not expected to be fluent in the use of a standard algorithm until the end of Grade 4.

According to Bloom's taxonomy, this work is both developmental and reinforcement.

Routines: Mentally add or subtract 10 or 100 to a given number between 100 and 900. Tell time to the nearest five minutes.

Future Learning

In grade 3, students will continue to add and subtract within 1000. They achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction. This is preparation for learning standard algorithms in grade 4.

Additional Findings

Principles and Standards for School Mathematics states, "The decisions that teachers make about problem solving opportunities influence the depth and breadth of students' mathematics learnig. Teachers must be clear about the mathematics they want their students to accomplish as they structure situation that are both problematic and attainable for a wide range of students." (pp. 118–119)

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At grade 2, composing and decomposing involves an extra layer of complexity beyond that of grade 1. This complexity manifests itself in two ways. First, students must understand that a hundred is a unit composed of 100 ones, but also that it is composed of 10 tens. Second, there is the possibility that both a ten and a hundred are composed or decomposed. For example, in computing 398 + 7 a new ten and a new hundred are composed. In computing 302 – 184, a ten and a hundred are decomposed.

Students may continue to develop and use special strategies for particular numerical cases or particular problem situations such as Unknown Addends. *Progressions, Number and Operations in Base Ten* (pp. 8–9)

Grade 2 Mathematics, Quarter 4, Unit 4.4

Collect, Represent, and Interpret Data

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Measure lengths to the nearest whole unit.
- Show data on a line plot.
- Represent data with up to four categories on a bar graph.
- Solve simple put-together, take-apart and compare problems using information from a graph.

Mathematical practices to be integrated

Model with mathematics.

- Display data in bar graph, picture graph or line plot.

Construct viable arguments and critique the reasoning of others.

- Use data to justify conclusions.

Look for and make use of structure.

- Make a line plot to display measurement data with appropriate magnitudes and spacing on the measurement scale used.

Essential questions

- How can you record data?
- How can you display the data you collect?
- What conclusions can you draw from the set of data?
- What questions can you create using the data?
- How can you determine what data to collect?
- How can you determine the best way to display the information/data collected?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	2.MD

Represent and interpret data.

2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

[4] See Glossary, Table 1.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

(1.MD.4) In first grade students organized, represented, and interpreted data with up to three categories; asked and answered questions about the total number of data points, how many in each category, and how many more or less in one category than in another. Their data come from categorical data sorting objects into categories. The focus was on bar graphs as a way to represent and analyze the data. They solved problems involving put-together, take-apart, compare data.

Current Learning

In second grade students draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a graph. They generate measurement data by measurements to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

Notable Connections: 1.MD.2 Length measurement and 2.MD.6 Number line

According to Bloom's taxonomy, student responses should be at the analyzing, applying, and understanding levels.

Routines:

- Add and subtract to 20 fluently.
- Tell time to nearest five minutes.
- Add and subtract with 1,000.

Future Learning

(3.MD.4) In grade 3, students will continue to generate data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by marking a line plot, where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters.

Notable Connections: 3.NF.2. Fractions on a number line.

Additional Findings

The graphic representation on page 115 of *The Atlas of Science Literacy,* shows the cognitive sequence for developing data analysis and graphic representation skills. The book also states, "The graphic display of numbers and relationships can be a powerful aid in discovering and communication patterns not easily seen in tables or equations" (pp. 11–115)

A Research Companion to Principles and Standards for School Mathematics states:

When students begin to learn how to create questions and collect and analyze their data they must be able to walk to fine lines. They must figure out how to make a statistical question specific enough so they can collect relevant data while making sure they do not trivialize their question in the process. Also they must learn to see the data they have created as separate in many ways from the real-world even they observed ;and not treat the data as numbers only. Students must be able to maintain a view of data as "numbers in context" and at the same time abstract the data from the context. Students must collect data to answer questions or solve a real-world problem (p. 195).

According to *Principles and Standards for School Mathematics,* in second grade all students should recognize the attributes of length, volume, weight, area, and time. They should understand measurable attributes of techniques, tools, and formulas to determine measurements (pp. 102–106).

Common Core Progressions gives examples of ways second grade students can generate a set of measurement data. For example "each student might measure the length of his or her arm in centimeters, or every student might measure the height of a statue in inches (p. 9).